CERTIFIED PUBLIC ACCOUNTANT

FOUNDATION LEVEL 1 EXAMINATION

F1.1: BUSINESS MATHEMATICS AND QUANTITATIVE METHODS

DATE: THURSDAY27, FEBRUARY 2025

MARKING GUIDE AND MODEL ANSWER

QUESTION ONE

Marking Guide

Question 1	Description	Marks	Total Marks
A	**Probability properties**		
	Property (0.5 Marks each for stating and 0.5 for explanation, Maximum 4)	4	**4**
b	**Computation of expected mean (profit)**		
	Computation of profit under machine A	1.5	
	Computation of profit under machine B	1.5	
	Advice	1	**4**
C	**Variance and standard deviation**		
	Formulas of variance and standard deviation (0.5 Marks each, Maximum)	1	
	Computation of variance	2	
	Calculation of standard deviation	1	**4**
D	**Venn Diagram**		
	Allocation of information in the Venn diagram (0.5 Marks each, Maximum 4)	4	
	Finding students who played all games	2	
	Finding students who played exactly one game	2	**8**
	Total Marks		**20**

Model Answer

a) Properties of probability as applied to Business Mathematics and Statistics.

1. **Probability limits/range property/ non – negativity**

The probability of an event must lie between 0 to 1. That is $0 \leq P(E) \leq P(E) \leq 1$. A probability close to 0 indicates that the event is not likely to occur. A probability close to one indicates that the event is most likely to happen.

2. **Complementary property**

The complement of an event A is the event that A does not occur. This is denoted as A^1 or A^c. If the probability of A is P(A) then the $P(A^1) = 1 - P(A)$. Example: The probability of a product being of acceptable quality is 2/3, therefore, the probability of a product not being of acceptable quality= 1 - 2/3 = 1/3.

3. **Exhaustive property/**

These are events whose sum of the probabilities of all possible outcomes in a sample space is one. Example: When a die is thrown once, the possible outcomes are 1, 2, 3, 4, 5, 6.P (1) + P (2) + P (3) + P (4) + P (5) + P (5) = 1/6 + 2/6 + 3/6 + 4/6 + 5/6 = 6/6 = 1. The summation of all probabilities is equal to 1

4. **Multiplicative property (Independent and dependent)**

These are events where the occurrence of one does not affect or influence the occurrence of the others. For example, the performance of female students and the male students in an

examination. For two independent events A and B the probability that both events occur is written as P (A and B) or P(A∩B). By the multiplication rule the independent events formula is given by P(A∩B) = P(A) x P(B) and $P(A/B) = \frac{P(A \cap B)}{P(B)}$ for conditional probability and dependent events

Example: If two events A and B are independent such that P(A)=0.35 and P(B)=0.6, find the probability that both events occur. P(A∩B) = P(A) x P(B) = 0.35 x0.6 = 0.21

5. **Additive Property (Mutually exclusive events and non – exclusive events)**

These are events that cannot occur at the same time on the same trial. For instance, A corporate ending a year with either "very high" or "very low" profits. A student "passing" or "failing" same exam. For two mutually exclusive events A and B, the probability that either A or B will occur is written as P(AUB).

This is obtained using the addition rule P(AUB) = P(A) + P (B) and P(AUB) = P(A) + P (B) – P(AnB)

8. Total probability

If events B_i are from the partition of the sample space, then for any set A, P(A)=$\sum P(A/B_i) \times (P_{B_I})$

b) Expected profit

Expected profit = Summation of (Profit × Profitability)

Expected profit = ∑ (Profit × Profitability)

Machine A; Expected profit = (900,000,000*0.6) + (100,000,000*0.4)

Expected profit = FRW 540,000,000 + 40,000,000 = **FRW 580,000,000**

Machine B; Expected profit = (850,000,000*0.7) + (250,000,000*0.3)

Expected profit = FRW 595,000,000 + 75,000,000 = **FRW 670,000,000**

The advice is that Mr. Murangira should invest in machine B because it has a higher expected profit.

b) Variance and standard deviation

Variance = ∑ [(X - μ)^{2}P(X)]

Mean μ = ∑[(X*P(X)]

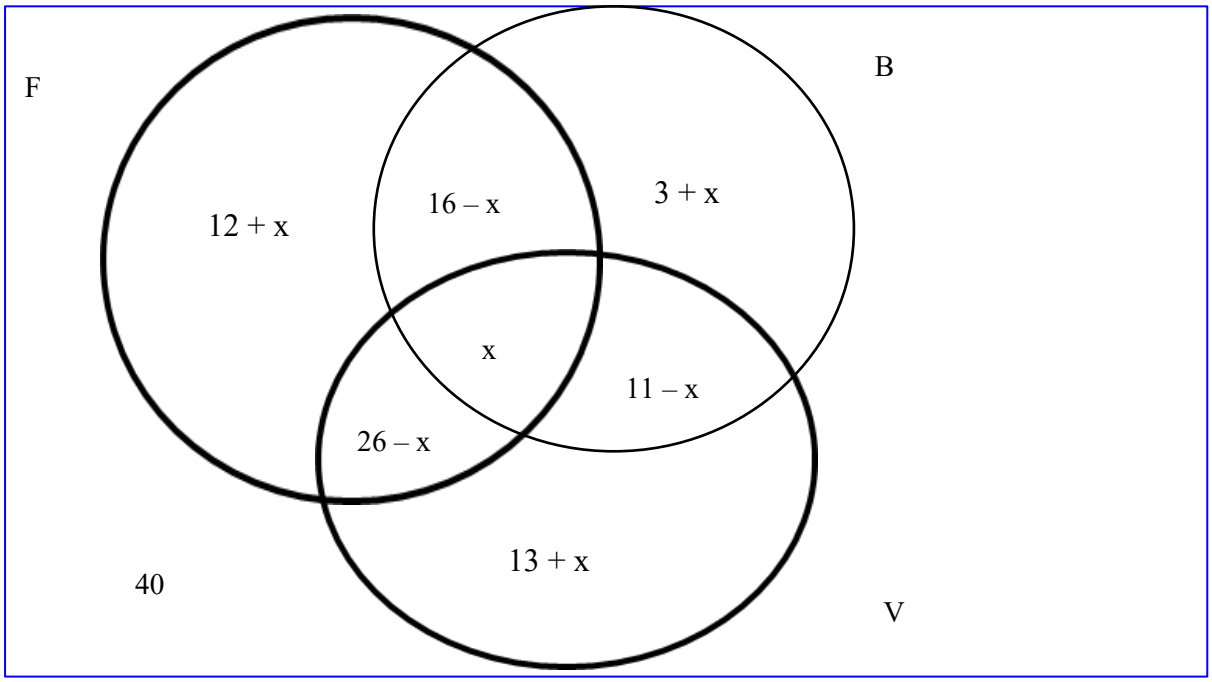
Standard deviation √ (Variance)

X	**0**	**1**	**2**	**3**	**4**	**Total**
P(X)	0.01	0.15	0.25	0.2	0.39	
Mean μ	0	0.15	0.5	0.6	1.56	**2.81**
(X - μ)	-2.81	-1.81	-0.81	0.19	1.19	
(X - μ)^2	7.8961	3.2761	0.6561	0.04	1.4161	
(X - μ)^{2}P(X)	0.078961	0.491415	0.164025	0.01	0.552279	**1.2939**
√(X - μ)^{2}P(X)						**1.1375**

Variance = $\sum [(X - \mu)^2 P(X)] = 1.2939$
Mean $\mu = \sum[(X*P(X)] = 2.81$
Standard deviation $\sqrt{}$ (Variance) = 1.1375

c) Venn Diagram

Let F represent the number of students who played football
Let B represent the number of students who played basketball
Let V represent the number of students who played volleyball
Let x represent the number of students who played all the 3 games
Let U represent the number of students who played none of the games
n(F) = 54, n(B) = 30, n(V) = 50, n(FnV) = 26, n(FnB) = 16 and n(BnV) = 11
n = 130, n(U) = 40
To get the number of students for each game;
F; 54 – 26 – 16 + x = 12 + x
B; 30 – 17 – 10 + x = 3 + x
V; 50 – 26 – 11 + x = 13 + x
Venn diagram



ii) The number of students who played all the games

To find the value of x, we add up all the values in the Venn diagram and equate to the total number of students

12 + x + 16 – x + 3 + x + 11 – x + 13 + x + 26 – x + x + 40 = 130
x + 121 = 130, x = 9
Those who played all the games were 9 students

iii) The number of students who played exactly one of the three games

12 + x + 3 + x + 13 + x = 12 + 9 + 3 + 9 + 13 + 9 = 55
The number of students who played exactly one of the three games was 55

QUESTION TWO

Marking Guide

Question 2	Description	Marks	Total Marks
a i)	**Standard error and Application of estimation**		
	Definition of standard error	1	
	Application of estimation	1	**2**
ii)	**Computation of population proportion**		
	Formula of population proportion	1	
	Substitution of figures in the formula	1	
	Computation of population proportion	2	**4**
B	**Normal probability distribution**		
I	Computation of P (x ≤ FRW 450,000) (1 Mark for z – score and 1 Mark for probability)	2	
Ii	Computation of P (FRW 550,000≤ x ≤ FRW 600,000)(1 Mark for z – score and 1 Mark for probability)	2	
Iii	Calculation of P (x ≥ FRW 650,000) (1 Mark for z – score and 1 Mark for probability)	2	**6**
C	**Binomial probability distribution**		
	Formula for binomial probability	1	
	Computation of probability of exactly five	2	
	Computation of probability of zero and one (1 Mark each)	2	
	Computation of probability of at least one	1	
	Computation of mean and standard deviation (1 Mark each)	2	**8**
	Total Marks		**20**

Model Answer

a)

i) Standard error of mean

Standard error of mean (SEM). It is a statistical measure that quantifies the amount of variation or dispersion of sample means around the population mean in estimation and sampling

Area of application

1. Inferential Statistics: SEM is used to make inferences about a population based on sample data, allowing researchers to estimate confidence intervals for the population mean.

2. **Hypothesis Testing:** It helps in determining the significance of sample means when testing hypotheses, particularly in t-tests and ANOVA. Sample Size Determination, Survey Methodology.

ii) Confidence interval of population proportion

Proportion = 850/1000 = 0.85, confidence interval of 95%, Z value = 2.58, P = 0.85, Q = 0.15, n = 1,000

Standard error of population proportion = $\sqrt{\frac{0.85*0.15}{1,000}} = \sqrt{0.1275}$ = 0.01129

Estimation of Population Proportion = P±Zscore *Standard error of proportion

Estimation of Population Proportion = P±$Z_{0.05}$ * $\sqrt{\frac{PQ}{n}}$

Estimation of Population Proportion = 0.85 ± (1.96*0.01129)

Estimation of Population Proportion = 0.85 ± 0.022

Estimation of Population Proportion = [0.828, 0.872,]

The population proportion is between 0.828 and 0.872 at 95% confidence interval

b) Normal probability distribution

μ= FRW 600,000 σ = FRW 50,000

i) P (x ≤ FRW 450,000)

$Z = \frac{x-\mu}{\sigma}$

P (Z = $\frac{\text{FRW 470,000} - \text{FRW 600,000}}{\text{FRW 50,000}}$) = P (Z≤ - 2.6) read from the z table

P (Z ≤ - 2.6) = 0.00466

i) P (FRW 550,000≤ x ≤ FRW 600,000)

P ($\frac{\text{FRW 550,000} - \text{FRW 600,000}}{\text{FRW 50,000}} \leq Z \leq \frac{\text{FRW 600,000} - \text{FRW 600,000}}{\text{FRW 50,000}}$)

P ($-1 \leq Z \leq 0$) = P (Z ≤ 0) - P (Z ≤ −1) = 0.50000 – 0.15866 = 0.34134

ii) P (x ≥ FRW 650,000)

P (Z ≥ $\frac{\text{FRW 680,000} - \text{FRW 600,000}}{\text{FRW 50,000}}$)

P (Z ≥ 1.6) = 1 – P (Z≤1.6) = 1 – 0.94520 = 0.0548

c) Binomial probability distribution

Probability of success, P = 0.03, Q = 1 – 0.03 = 0.97, n = 20

i) P (x = 5)

P (x) = $\frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$

P (x = 5) = $\frac{20!}{5!(20-5)!} 0.03^5 * 0.97^{20-5}$

P (x = 5) = $\frac{20!}{5!15!} 0.03^5 * 0.97^{15}$

P (x = 5) = $\frac{20*19*18*17*16*15!}{5!15!} 0.03^5 * 0.97^{15}$

P (x = 5) = = $\frac{20*19*18*17*16*15!}{5!15!} * 0.0000000243 * 0.2539 = 0.000239$

ii) P (x≤1) = 1 -[P(x=0) + P(x=1)]

P (x≤1) = 1 - [($\frac{20!}{0!(20-0)!} 0.03^0 * 0.97^{20-0}$)]

P (x≤3) = 1 – [0.5438]

P (x≤3) = 1 – 0.8802 = 0.4562

iii) The expected number of expired drugs and standard deviation

Mean = n × p where n is the sample size, p is the probability of success. n = 20, p = 0.03

Mean = 20 * 0.03 = 0.6

Standard deviation = $\sqrt{n \times p \times q}$ =$\sqrt{20 \times 0.03 \times 0.97}$ = 0.7629

QUESTION THREE

Marking Guide

Question 3	Description	Marks	Total Marks
a	**Types of capital investment problems**		
	Types (1 Mark each, maximum 2)		**2**
b	**NPV through discounted cash flows**		
	Computation of present values (0.5 Marks each, maximum 3)	3	
	Computation of NPV	1	
	Advice	2	**6**
C	**Hypothesis Testing**		
	Stating the hypotheses	1	
	Finding the critical value from the table	1	
	Calculation of the standardized value	2	
	Decision	2	**6**
D	**Index Numbers**		
	Computation of totals (0.5 Marks each, maximum 2)	2	
	Computation of Laspyere's price index (0.5 Marks for formula and 0.5 marks for computation)	1	
	Computation of Paasche's price index (0.5 Marks for formula and 0.5 marks for computation)	1	
	Computation of Fischer's price index (0.5 Marks for formula and 0.5 marks for computation)	1	
	Interpretation of Fischer's price index	1	**6**
	Total Marks		**20**

Model Answer

a) Types of capital investment problems

1. The replacement of, or improvement in, existing assets by more efficient plant and equipment (often measured by the estimated cost savings).
2. The expansion of business facilities to produce and market new products (measured by the forecast of additional profitability against the proposed capital investment).
3. Decisions regarding the choice between alternatives where there is more than one way of achieving the desired result.
4. Decisions whether to purchase or lease assets.

b) Discounted cash flows

Year	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
Cash flow (FRW'000')	-850,000	150,000	200,000	230,000	280,000	320,000
Discount factor (10%)	1.000	0.909	0.826	0.751	0.683	0.621
Present Values (FRW'000')	(850,000)	136,350	165,200	172,730	191,240	198,720
Net Present Value (FRW'000')	**14,240**					

The Net Present Value is FRW 14,240,000

Decision: The project should be accepted.

c) Hypothesis testing

n =100, μ = 10 hours, s = 1.2 hours, $\bar{x}$=8 hours, Level of significance = 5%

1. Statement of hypotheses
   Null hypothesis; H_0: μ=10 hours
   Alternative hypothesis; H_A: μ<8 hours
2. Statement of level of significance to be used
   The level of significance is 5%
3. Statement about the test statistic
   The test statistic is testing sample mean, $\overline{X}$= 8 hours
4. The critical value of the one tailed test at 5% level of significance is – 1.65(left-sided)
5. The standardized value of the sample mean
   $Z = \frac{\bar{x}-\mu}{S_{\bar{x}}}$
   $S_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{1.2}{\sqrt{100}} = 0.12$
   $Z = \frac{8-10}{0.12} = -16.67$
6. Decision: Since – 16.67 is less than – 1.65, we reject the null hypothesis but accept the alternative hypothesis at 5% level of significance i.e. the motorcycle battery did not last10 hours before being recharged.

d) Index numbers

	2021	2023

Commodity	Price (FRW) PO	Quantity (Kgs) QO	Price (FRW) P1	Quantity (Kgs) Q1
Maize	1,500	100	1,600	180
Rice	1,900	150	2,100	200
Beef	3,200	200	3,400	150
Chicken	4,500	250	4,800	260

Commodity	P0Q0	P1Q1	P0Q1	P1Q0
Maize	150,000	288,000	270,000	160,000
Rice	285,000	420,000	380,000	315,000
Beef	640,000	510,000	480,000	680,000
Chicken	1,125,000	1,248,000	1,170,000	1,200,000
	2,200,000	**2,466,000**	**2,300,000**	**2,355,000**

Laspyre's Price Index

Laspyre's Price Index = $\frac{\sum P_1Q_0}{\sum P_0Q_0} \times 100$

Laspyre's Price Index = $\frac{\mathbf{2,355,000}}{\mathbf{2,200,000}} \times 100$=**107.05**

Paasche's Price Index

Paasche's Price Index = $\frac{\sum P_1Q_1}{\sum P_0Q_1} \times 100$

Paasche's Price Index = $\frac{2,466,000}{2,300,000} \times 100$=**107.22**

Fischer's Price Index

Fischer's Price Index = $\sqrt{\frac{\sum P_1Q_0}{\sum P_0Q_0} \times \frac{\sum P_1Q_1}{\sum P_0Q_1}} \times 100$

Fischer's Price Index = $\sqrt{\frac{2,355,000}{2,200,000} \times \frac{2,466,000}{2,300,000}} \times 100$=**107.13**

Interpretation: There was an increase of 7.13% of the basket of commodities from 2021 to 2023.

QUESTION FOUR

Marking Guide

Question 4	Description	Marks	Total Marks
A	**Difference between random sampling and non – random sampling and types of non – probability sampling**		
	Definition of random sampling	1	
	Definition of non - random sampling	1	
	Types of non – probability sampling (1 Mark each, maximum 2)	2	**4**
B	**Rules of differentiation**		
	Rules (1 Mark each, maximum 4)		**4**
C	**Function**		
	Computation of f(x) for given values of x (0.5 Marks each, maximum 2)	2	
	Coordinates drawn on the graph (0.5 Marks each, maximum 2)	2	**4**
	Simultaneous equation		
	Computation of x	2	
	Computation of y	2	**4**
d	**Inverse Function**		
	Formula for inverse	1	
	Computation of the determinant	1	
	Computation of the cofactor	1	
	Computation of inverse	1	**4**
	Total Marks		**20**

Model Answer

a) Difference between random sampling and non–random sampling

Random sampling is also referred to as probability sampling. In this method, each item of the population under consideration has an equal chance of being selected as part of the sample. Each item (subject) in the population is selected independent of the rest of the items. While

Non-random sampling is a method used to select elements from the population based on other specific criteria or convenience, rather than random selection. This is used when the random sampling techniques are impossible or not practical to use. This method comprises of cluster and quota sampling techniques.

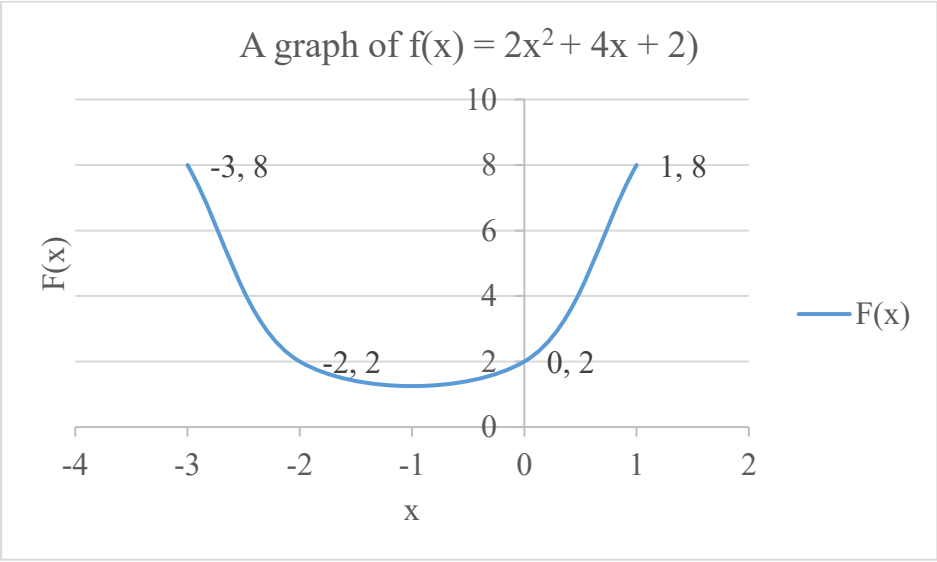
Types of non – probability sampling

1. **Purpose sampling**
   This is also called a deliberate or judgment sampling. In this when the researcher deliberately selects certain units for study from the universe is known as purpose sampling.
2. **Quota sampling**
   In this method, the population is stratified on some basis, preferably on the characteristics of population under study. After this the number of sample units to be selected from each stratum is decided by the researcher in advance. This number is known as Quota which may be fixed is fixed according to specific characteristics such as income group, sex, occupation, etc. the investigator usually applies their judgment in choice of sample and try to complete quota assigned form each stratum.
3. **Convenience Sampling**
   It is known as unsystematic, careless, accidental or opportunistic sampling. Under this sample is selected according to convenience of the investigator. This may have the base of availability of data, accessibility of units, etc.

b) Rules of differentiation

1. Power functions rule. Where the function is= x^2, the derivative $f(x)' = nx^{n-1}$
2. Chain Rule. This deals with the derivative of a function of a function such as $y = (px+q)^n$
3. Product rule of functions. When the function is a product of two or more functions: it takes the form mn - f(x)g(x). Let m and n represent functions of x and y = mn
4. Quotient rule. This deals with division of two or more functions.
5. Addition and subtraction rule
6. Constant rule
7. Exponential rule
8. Logarithmic rule

c) Function $f(x) = 2x^2 + 4x + 2$ for x = - 3, -2, 0, 1



X		-3	-2	0	1
F(x)		8	2	2	8

d) Simultaneous equation

Let the cost of maize be x and the cost of beans be y

200x + 300y = 1,300…….. (1)

300x + 200y = 1,200…….. (2)

Solving by elimination method, let's eliminate x

300(200x + 300y = 1,300)

200(300x + 200y =1,200)

60,000x + 90,000y = 390,000

60,000x + 40,000y = 240,000

50,000y = 150,000

y = 3

Substitute y = 3 into equation 1

200x + 300(3) = 1,300

200x + 900 = 1,300

200x = 1,300 – 900

200x = 400

x =2

It will cost FRW2/kg for maize and FRW3/kg for beans.

e) Inverse function

200x + 300y = 1,300

300x + 200y = 1,200

Multiplication by dividing by 100

2x + 3y = 13

3x + 2y = 12

$\begin{bmatrix}2 & 3\\3 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}13\\12\end{bmatrix}$

Arranging in matrix form; A = $\begin{bmatrix}2 & 3\\3 & 2\end{bmatrix}$

Formula for inverse; $A^{-1} = \frac{1}{\det A} \times A^{c}$

Computing the determinant; det A = (2 × 2) - (3 × 3) = - 5

Computing the cofactor; $A^c = \begin{bmatrix} 2 & -3 \\ -3 & 2 \end{bmatrix}$

Computation of inverse; $A^{-1} = -\frac{1}{5}\begin{bmatrix} 2 & -3 \\ -3 & 2 \end{bmatrix}$ Or $\begin{bmatrix} -2/5 & 3/5 \\ 3/5 & -2/5 \end{bmatrix}$

QUESTION FIVE

Marking Guide

Question 5	Description	Marks	Total Marks
A	**Difference between central tendency and dispersion**		
	Central tendency definition	1	
	Dispersion definition	1	**2**
B	**Computation of Mean, Median & Mode**		
	Mean		
	Total of FX	1	
	Formula of mean	0.5	
	Calculation of mean	0.5	**2**
	Median		
	Computation of CF (0.5 Marks each, Maximum 2.5)	2.5	
	Formula of median	0.5	
	Calculation of median	1	**4**
	Mode		
	Formula of mode	0.5	
	Calculation of mode	0.5	**1**
C	**Histogram**		
	Drawing of bars (1 Mark each, Maximum 5)	5	
	Labelling axes (1 Mark each, Maximum 2)	2	**7**
D	**Branches of statistics**		
	Explanation of descriptive statistics	1	
	Explanation of inferential statistics	1	**2**
E	**Permutation**		
	Definition of permutation		**2**
	Total Marks		**20**

a) Difference between measures of central tendency and measures of variation or dispersion or spread

Measures of central tendency are methods (or values) used to describe the middle or a central characteristic of a set of data that best represents all the numbers in the sample or population. They show the average position of a given distribution of data. They are values which lie between the two extreme observations in the distribution that are used to summarize data by trying to find one number. In general, they are referred to as averages. In statistics, an average is defined as the number that measures the central tendency of a given set of numbers.

While

Dispersion is the degree or extent of spread of items in a distribution around a measure of central tendency. A measure of dispersion indicates the extent to which the individual observations differ from the mean or from any other measure of central tendency. The measures of dispersion are also called the measure of variation or measures of spread. The measure of dispersion is said to be absolute if it is expressed in the units of the variables. However, if it is expressed in the form of a co-efficient, ratio or percentage then it is said to be a relative measure of dispersion.

Aspect	**Measures of Central Tendency**	**Measures of Dispersion/Variation**
Definition	Represent the **central point** of a data set	Represent the **spread** or **variability** of data
Purpose	Identify the **typical value** in a data set	Show how **spread out** the values are from the **central value**
Common Measures	- **Mean** (Average)	- **Range** (Difference between max and min values)
	- **Median** (Middle value when data is ordered)	- **Variance** (Average squared deviation from the mean)
	- **Mode** (Most frequent value)	- **Standard Deviation** (Square root of variance)
Example	Averages scores of students in a test	How much test scores **vary** among students

Aspect	Measures of Central Tendency	Measures of Dispersion/Variation
Application	Useful for understanding **general performance**	Useful for understanding **consistency** and **risk**

b) Calculations of mean, median and mode

Scores	Frequency (F)	Class mid points (X)	FX	Cumulative Frequency (CF)
0 – 20	7	10	70	7
20 – 40	12	30	360	19
40 – 60	18	50	900	37
60 – 80	10	70	700	47
80 – 100	3	90	270	50
	50		**2300**	

Mean

Mean = $\frac{\sum FX}{\sum f}$ or $\frac{\sum FX}{n}$

Mean = $\frac{2300}{50}$ = 46

Median

Median = $L + \frac{(\frac{N}{2} - CFBmc)}{Fmc} \times C$

Where L is the class boundary of the median class, N is the sample size, CFBmc is the cumulative frequency before the median class, Fmc is the frequency of the median class, C is the class width

L = 40, N/2 = 50/2 = 25, CFBmc = 7, Fmc = 12, C = 20

Median = $40 + \frac{(25-19)}{18} \times 20$ = 46.67

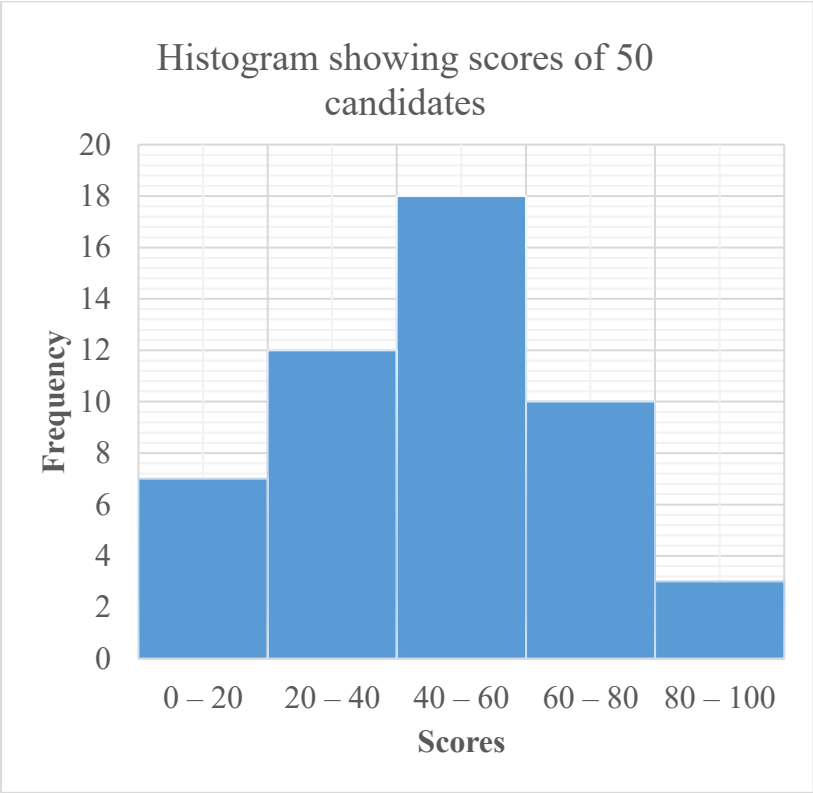
Mode

Mode = $L + \frac{(F_1 - F_0)}{(2F_1 - F_2 - F_0)} \times C$

Where L is the class boundary of the modal class, C is the class width, F_0 is frequency before a modal class, F_1 is the class of the modal class and F_2 is frequency after a modal class.

Mode = $40 + \frac{(18-12)}{(2*18-10-12)} \times 20$ = 48.57

c) Histogram



d) Branches of statistics

There are mainly two branches of statistics. Descriptive statistics and Inferential statistics

1. Descriptive statistics: This is a branch of statistics which deals with methods of collection of data, its presentation and organization in various forms, such as distribution tables, graphs (e.g., ogive, Lorenz curves, etc.), diagrams (e.g., pie charts) and finding measures of central tendency and measures of dispersion or spread which are used in the description of data. Managers, CEOs. etc. make use of descriptive statistics in presenting their annual reports, financial accounts and bank statements. Descriptive statistics is used to present the data in an understandable way, so that a meaningful description can be made.
2. Inferential or predictive statistics: This is a branch of statistics which deals with techniques used for analysis of data, making estimates that lead to predictions and drawing conclusions or inferences from limited information taken on sample basis and testing the reliability of the estimates or predictions. Inferential statistics is used to make comparisons or predictions about a larger group, known as population, using information gathered about a small part of that population called a sample.

e) Permutation

Permutation is an arrangement of n distinct items in a specific order. AB and BA are different arrangements. These arrangements are known as permutations. The number of possible arrangements when r objects are taken from a total of n distinct objects is given by the permutation formula

$Permutation = \frac{n!}{(n-r)!}$ Where n is the number of total objects and r is the objects to choose or taken from the total. Only and only if r = n

QUESTION SIX

Marking Guide

Question 6	Description	Marks	Total Marks
A	**Decision Theory**		
i)	**Maximax** (0.5 Marks for decision and 0.5 for the reason)	1	
ii)	**Maximin** (0.5 Marks for decision and 0.5 for the reason)	1	**2**
iii)	**Minimax**		
	Computation of regrets (0.5 Marks each, Maximum 4.5)	4.5	
	Decision	0.5	**5**
Iv	**Hurwitz**		
	Formula for Hurwitz	0.5	
	Calculation of payoff totals (0.5 Marks each, Maximum 1.5)	1.5	
	Decision	1	**3**
B	**Assignment**		
	Steps 1 to 4 (2 Marks each, Maximum 8)	8	
	Step 5	1	
	Optimal assignment	0.5	
	Total minimum distance	0.5	**10**
	Total Marks		**20**

Model Answer

a) Decision theory

	Favourable Market (FRW 000)	Moderate Market (FRW 000)	Unfavourable market (FRW 000)	Maximum	Minimum
Real Estates	1,500	850	500	**1,500**	500
Agri-business	1,300	900	400	1,300	400

Bonds	1,000	750	600	1,000	**600**
				Maximax	**Maximin**

i) Maximax

Decision: The decision under the Maximax criterion is to invest in **Real Estates** because it has the maximum of the maximum payoff of **FRW 1,500,000**

ii) Minimax

Decision: The decision under the Maximin criterion is to invest in **Bonds** because it has the maximum of the minimum payoff of **FRW 600,000**

iii) Minimax

Regret table

	Favourable Market (FRW 000)	Moderate Market (FRW 000)	Unfavourable market (FRW 000)	Maximum
Real Estates	0	50	100	**100**
Agri-business	200	0	200	200
Bonds	500	150	0	500
				Minimax

Decision: The decision under the Minimax criterion is to invest in **Real Estates** because it has the minimum of the maximum payoff of **FRW 100,000**

iv) Hurwitz

$\alpha = 0.7$

Payoff = (α * maximum) + ((1- α) * maximum)

Real Estates; Payoff = (0.7 * 1,500,000) + (0.3 * 500,000) = **FRW 1,200,000**

Agri – business; Payoff = (0.7 * 1,300,000) + (0.3 * 400,000) = FRW 790,000

Bonds; Payoff = (0.7 * 1,000,000) + (0.3 * 600,000) = FRW 520,000

Decision: The decision under the Hurwitz criterion is to invest in **Real Estates** because it has the highest payoff of **FRW 1,200,000**

b) Assignment Problem

Distribution / **Provinces**	Centre I	Centre II	Centre III	Centre IV	Centre V
Kigali City	180	150	195	210	220
Northern	155	140	150	180	190
Southern	110	80	125	140	155
Eastern	40	40	70	70	130
Western	45	25	60	70	95

Step 1: Subtract the minimum values in the rows

Distribution / Provinces	Centre I	Centre II	Centre III	Centre IV	Centre V
Kigali City	30	0	45	60	70
Northern	15	0	10	40	50
Southern	30	0	45	60	75
Eastern	0	0	30	30	90
Western	20	0	35	45	85

Since the number of lines is less than the number of columns/rows, we go for step 2

Step 2: Subtract the minimum values in the columns

Distribution / Provinces	Centre I	Centre II	Centre III	Centre IV	Centre V
Kigali City	30	0	35	30	20
Northern	15	0	0	10	0
Southern	30	0	35	30	25
Eastern	0	0	20	0	40
Western	20	0	25	15	35

Since the number of lines is less than the number of columns/rows, we go for step 3

Step 3: Subtract the lowest value from all the values not covered by the line and add them to the values in the intersection

Distribution / Provinces	Centre I	Centre II	Centre III	Centre IV	Centre V
Kigali City	15	0	20	15	5
Northern	15	0	0	10	0
Southern	15	0	20	15	10
Eastern	0	15	20	0	40
Western	15	0	10	0	20

Since the number of lines is less than the number of columns/rows, we go for step 4 where step 3 is repeated

Step 4: Modification II - Repeat the step 3 for another least value obtained

Distribution / **Provinces**	Centre I	Centre II	Centre III	Centre IV	Centre V
Kigali City	10	0	15	15	0
Northern	15	5	0	15	0
Southern	10	0	15	15	5
Eastern	0	20	20	5	40
Western	0	0	5	0	15

Since the number of lines is equal to the number of columns/rows, we go for step 5 for optimal assignment

Step 5: Optimal assignment

Distribution / **Provinces**	Centre I	Centre II	Centre III	Centre IV	Centre V
Kigali City	10	0	15	15	0
Northern	15	5	0	15	0
Southern	10	0	15	15	5
Eastern	0	20	20	5	40
Western	0	0	5	0	15

Assignment	**Total Minimum distance**
Kigali City - Centre V	220
Northern - Centre III	150
Southern - Centre II	80
Eastern - Centre I	40
Western - Centre IV	70
	560

The minimum total distance is 560 kilometres

QUESTION SEVEN

Marking Guide

Question 7	Description	Marks	Total Marks
a	**Correlation coefficient**		
	The totals (0.5 Marks each, Maximum 3)	3	
	Formula	1	
	Computation of coefficient of correlation	2	**6**
b	**Forecasting using regression**		
	Formula for b	1	
	Computation of b	2	
	Formula for a	1	
	Computation of a	1	
	Forecasted sales	1	**6**
c	**Network Analysis**		
	Each activity drawn (0.5 Marks each, Maximum 5)	5	
	Critical path	1	
	Project duration	1	
	Definition of network analysis	1	**8**
	Total Marks		**20**

Model Answer

a) Correlation coefficient

Months	**Sales (FRW '000')** **y**	**Advertising Expenditure (FRW '000')** **x**	**xy (FRW '000')**	**x^2 (FRW '000')**	**y^2 (FRW '000')**
1	2,800	800	2240000	640000	7840000
2	3,600	1,200	4320000	1440000	12960000
3	4,600	1,800	8280000	3240000	21160000
4	6,800	2,600	17680000	6760000	46240000
5	8,200	3,600	29520000	12960000	67240000
6	9,800	4,800	47040000	23040000	96040000
6	**35,800**	**14,800**	**109,080,000**	**48,080,000**	**251,480,000**

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n\sum x^2 - (\sum x)^2][n\sum y^2 - (\sum y)^2]}}$$

$$r = \frac{6(109{,}080{,}000) - (14{,}800)(35{,}800)}{\sqrt{[6*48{,}080{,}000) - (14{,}800*14{,}800)][6*251{,}480{,}000 - (35{,}800*35{,}800)]}}$$

$$r = \frac{654,480,000 - 529840000}{\sqrt{[288,480,000 - 219,040,000][1,508,880,000 - 1,281,640,000]}}$$

$$r = \frac{124,640,000}{\sqrt{[69,440,000][227,240,000]}}$$

$$r = \frac{124,640,000}{125,616,661}$$

$r = 0.99$

Interpretation: There is a strong direct correlation between sales and advertising expenditure.

b) Forecasting using regression

Months	**Sales (FRW '000')** **y**	**Advertising Expenditure (FRW '000')** **x**	**xy (FRW '000')**	**x^2 (FRW '000')**
1	2,800	800	2240000	640000
2	3,600	1,200	4320000	1440000
3	4,600	1,800	8280000	3240000
4	6,800	2,600	17680000	6760000
5	8,200	3,600	29520000	12960000
6	9,800	4,800	47040000	23040000
6	**35,800**	**14,800**	**109,080,000**	**48,080,000**

The regression line/equation is given by $y = a + bx$

b is the slope and a is the intercept. We have to look for these constants

$$b = \frac{n(\sum xy) - (\sum x)(\sum y)}{n\sum x^2 - (\sum x)^2} = \frac{(6 * 109,080,000) - (35,800 * 14,800)}{6 * 48,080,000 - (14,800)^2} = \frac{654,480,000 - 529840000}{288,480,000 - 219,040,000}$$

b = 1.79

$$a = \frac{\sum y - b\sum x}{n} = \frac{(35800) - (1.79 * 14,800)}{6} = \textbf{FRW 1551.33}$$

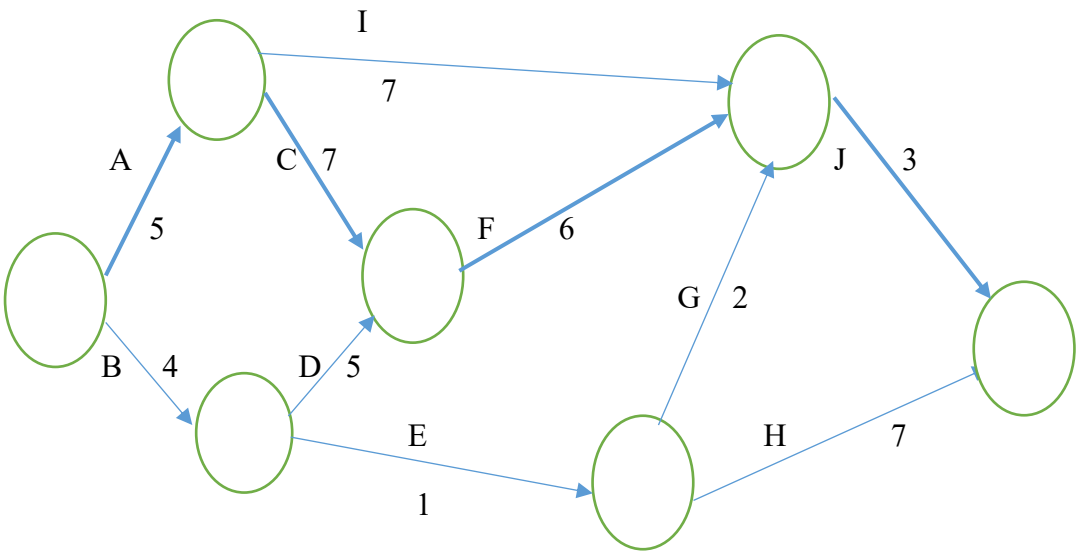
The regression becomes y = FRW 1551.33 + 1.79 x

For the advertising expenditure of FRW 5,200,000, sales will be?

x = FRW 5,200,000, substitute into the equation above

Sales, y=1539.17+1.79(5200)
Sales, y = FRW 10,872,810

i) Activity network diagram



ii) Critical path

A – I – J; 5 + 7 + 3 = 15 Weeks
A – C – F – J; 5 + 7 + 6 + 3 = 21 Weeks
B – D – F – J; 4 + 5 + 6 + 3 = 18 Weeks
B – E – G – J; 4 + 1 + 2 + 3 = 10 Weeks
B – E – H; 4 + 1 + 7 = 12 Weeks

The critical path is **A – C – F – J**

iii) Duration for the project

The project duration is 21 weeks

iv) Network analysis

Network analysis is the organized application of systematic, logical planning, scheduling and controlling practical situations, where many separate tasks can take place simultaneously or consecutively, such that it is difficult to establish the relationship between the separate jobs. The technique can be applied to any purposeful chain of events involving the use of time, labour and physical resources. It is usually related to large industrial or commercial projects of a complex nature where the scale of operation gives rise to correspondingly greater financial and administrative problems.

End of the Marking guide and Model Answers