



CERTIFIED PUBLIC ACCOUNTANT
FOUNDATION LEVEL 1 EXAMINATION
F1.1: BUSINESS MATHEMATICS AND QUANTITATIVE
METHODS
MODEL ANSWERS AND MARKING GUIDE
DATE: AUGUST 2025

QUESTION ONE

Marking Guide

| Question One | Description | Marks |
|--------------|---|-----------|
| a) | Simplex | |
| | Definition of decision variables | 1 |
| | Objective function | 0.5 |
| | Constraints; 0.5 marks each | 1 |
| | Initial tableau (correct solution 0.5 marks each) | 3 |
| | Table 2 (correct solution 0.5 marks each) | 3 |
| | Table 3 (correct solution 0.5 marks each) | 3 |
| | Table 4 (correct solution 0.5 marks each) | 3 |
| | Optimal solution; 0.5 marks each | 1 |
| | Maximum profit | 0.5 |
| | Maximum marks | 16 |
| | | |
| b) | Limitations of linear programming | |
| | Limitations (1 mark each) | 4 |
| | Total Marks | 20 |

Model Answer

a) Simplex model method

Linear programming model

Defining decision variables

Let x, y and z be the number of chairs, tables and cupboards to be produced respectively

Objective function

The objective is to maximize profit

$$\text{Profit, } P = 7x + 8y + 10z$$

Constraints

The question has only two constraints;

$$\text{Labour time constraint: } 2x + 3y + 2z \leq 1,000$$

$$\text{Machine time constraint: } x + y + 2z \leq 800$$

Non – negativity constraint: $x \geq 0$ and $y \geq 0$

Incorporation of slack variables

$$2x + 3y + 2z + S1 = 1,000$$

$$x + y + 2z + S2 = 800$$

$$P - 7x - 8y - 10z = 0$$

Initial simplex tableau

| | x | y | z | s₁ | s₂ | p | Solution | Ratio |
|----------------------|----------|----------|----------|----------------------|----------------------|----------|-----------------|---------------|
| s₁ | 2 | 3 | 2 | 1 | 0 | 0 | 1,000 | 1,000/2 = 500 |
| s₂ | 1 | 1 | 2 | 0 | 1 | 0 | 800 | 800/2 = 400 |
| p | -7 | -8 | -10 | 0 | 0 | 1 | 0 | |

Column z is the pivot column, row 2 is the pivot row well as 2 is the pivot number. We therefore have to change 2 to 1 by dividing the whole row by 2

Table 2

| | x | y | z | s₁ | s₂ | p | Solution | | |
|----------------------|----------|----------|----------|----------------------|----------------------|----------|-----------------|----------------------------------|---------------|
| s₁ | 1 | 2 | 0 | 1 | -1 | 0 | 200 | -2R ₂ +R ₁ | 200/2 = 100 |
| Z | ½ | ½ | 1 | 0 | ½ | 0 | 400 | R ₂ /2 | 400/1/2 = 800 |
| P | -2 | -3 | 0 | 0 | 5 | 1 | 4,000 | 10R ₂ +R ₃ | |

We start again by checking for the pivot column, pivot row and the pivot number

The pivot column is y since – 3 is the most negative number in the last row. The pivot row will be the first row since 100 is the smallest non negative value. The pivot number will be 2 which we have to convert to 1 using row elementary operation

Table 3

We start again by checking for the pivot column, pivot row and the pivot number

| | x | y | z | s₁ | s₂ | p | Solution | | |
|----------|----------|----------|----------|----------------------|----------------------|----------|-----------------|----------------------------------|-----------------|
| Y | ½ | 1 | 0 | ½ | -½ | 0 | 100 | R ₁ /2 | 100/1/2 = 200 |
| z | ¼ | 0 | 1 | -1/4 | ½ | 0 | 350 | -½R ₁ +R ₂ | 350/1/4 = 1,400 |
| p | -½ | 0 | 0 | 3/2 | 7/2 | 1 | 4,300 | 3R ₁ +R ₃ | |

We start again by checking for the pivot column, pivot row and the pivot number since there is still a negative number in the last row

We take the first row since it has the smallest non negative number

Table 4

| | x | y | z | s1 | s2 | p | Solution | |
|----------|----------|----------|----------|-----------|-----------|----------|-----------------|-----------------|
| X | 1 | 2 | 0 | 1 | -1 | 0 | 200 | $R_1 \cdot 1/2$ |
| Z | 1 | $-1/2$ | 1 | $-1/2$ | $3/2$ | 0 | 300 | $-1/4R_1 + R_2$ |
| P | 0 | 1 | 0 | 2 | 3 | 1 | 4,400 | $1/2R_1 + R_3$ |

Optimal solution from the simplex tableau is 200 chairs and 300 cupboards should be produced to obtain a maximum profit of FRW 4,400.

b) Limitations of linear programming.

Limitations

- The objective function and the constraints in some problems are not linear. LPP under non-linear condition usually results in a wrong solution.
- The parameters in the LP model are assumed to be constant. But in real life situation they are neither constant nor deterministic.
- LP is applicable to only static situations since it does not take into account the effect of time. LPP may get fractional valued answers for the decision variables, however only the integer values of the variables are rational and logical.
- LPP deals with problems that have a single objective. Some real-life problems may have more than one objective.
- LP cannot be used effectively for large scale problems i.e. those that may have more than two decision variables, this leads to enormous computational challenges.
- **Ignores Uncertainty:** It does not account for changes in resource availability, demand, or cost over time
- **Requires Accurate Data:** The model's reliability depends heavily on precise input data; any errors can lead to misleading results.
- **Cannot Handle Non-Quantifiable Factors:** Qualitative aspects like employee morale, customer satisfaction, or brand image are not considered.

QUESTION TWO

Marking Guide

| Question Two | Description | Marks |
|--------------|---|-----------|
| a) | Gasabo Ltd | |
| i) | Derivation of total cost function | 1 |
| | Derivation of total revenue function | 1 |
| | Derivation of profit function | 1 |
| ii) | Calculation of total profit | 1 |
| iii) | Calculation of breakeven units | 1 |
| | Calculation of breakeven sales revenue | 1 |
| | | |
| b) | Maniraguha Company | |
| i) | Revenue function and Profit function | 1 2 |
| | Marginal profit function | 2 |
| | Output that maximizes profit | 2 |
| ii) | Maximum profit | 1 |
| iii) | Derivation of marginal cost function | 1 |
| | Derivation of marginal revenue function | 1 |
| iv) | Graph (1 mark each coordinate) | 4 |
| | Total Marks | 20 |

Model Answer

a) Gasabo Ltd (GL)

i) **Derivation of the total cost, total revenue and profit functions**

Solution

Derivation of functions

Total cost = Variable cost + Fixed cost

Variable cost = 6,000x Fixed cost = 120,000, Let quantity be x

Total cost function $C(x) = 6,000x + 120,000$

Total revenue = Price * Quantity

Price is FRW 7,000 and Quantity is x

Therefore, Total revenue function $R(x) = 7,000x$

Profit = Total revenue – Total cost

Profit function $P(x) = 7,000x - (6,000x + 120,000)$

Profit function $P(x) = 1,000x - 120,000$

ii) Calculation of the total profit of producing 2,500 bottles of juice.

Profit for 2,500 bottles

Profit function $P(x) = 1,000x - 120,000$

Profit function $P(2500) = 1,000 \cdot 2500 - 120,000 = \text{FRW } 2,380,000$

iii) Computation of the number of bottles of juices GL should produce to break even and find the breakeven in sales revenue.

Break even units (bottles) and sales revenue (FRW)

Break-even (units)

Break – even is where profit = 0

$P = 1,000x - 120,000$

$1,000x - 120,000 = 0$

$1,000x = 120,000$

$x = 120$. The company will breakeven at 120 bottles

Break-even sales revenue

Break-even sales revenue = 120 bottles * FRW 7,000 = FRW 840,000

b) Maniraguha company

i) Finding the profit function, marginal profit function and out that maximizes profit.

Profit function

Profit function $P(x) = \text{Total Revenue} - \text{Total cost}$

Total Revenue $R(x) = \text{Price} * \text{Quantity}$

Total Revenue $R(x) = (100 - 2x) x = 100x - 2x^2$

Profit function $P(x) = 100x - 2x^2 - (20x + 0.5x^2 + 500)$

Profit function $P(x) = 100x - 2x^2 - 20x - 0.5x^2 - 500$

Profit function $P(x) = 80x - 2.5 x^2 - 500$

Marginal Profit function

We take the derivative of profit function

Marginal Profit function; $P(x)' = (80x - 2.5 x^2 - 500)'$

Marginal profit function $P(x)' = 80 - 5x$

Output that maximizes profit

Equate the marginal profit to zero

Marginal profit function $P(x)' = 0$

$$80 - 5x = 0$$

$$5x = 80, x = 16$$

Output that maximises profit is 16,000 bags

ii) Computation of the maximum profit.

Maximum profit is obtained by substituting x into the profit function

$$x = 16$$

$$P(x) = 80x - 2.5 x^2 - 500$$

$$P(16) = 80*16 - 2.5 (16)^2 - 500 = 1,280 + 640 - 500 = 140$$

Maximum profit is FRW 140,000

iii) **Derivation of marginal cost and marginal revenue functions for Maniraguha company.**

Marginal cost

Marginal cost is derived from total cost

$$C(x) = 20x + 0.5x^2 + 500 = 20 + 2(0.5)x$$

$$C'(x) = 20 + x$$

Marginal revenue

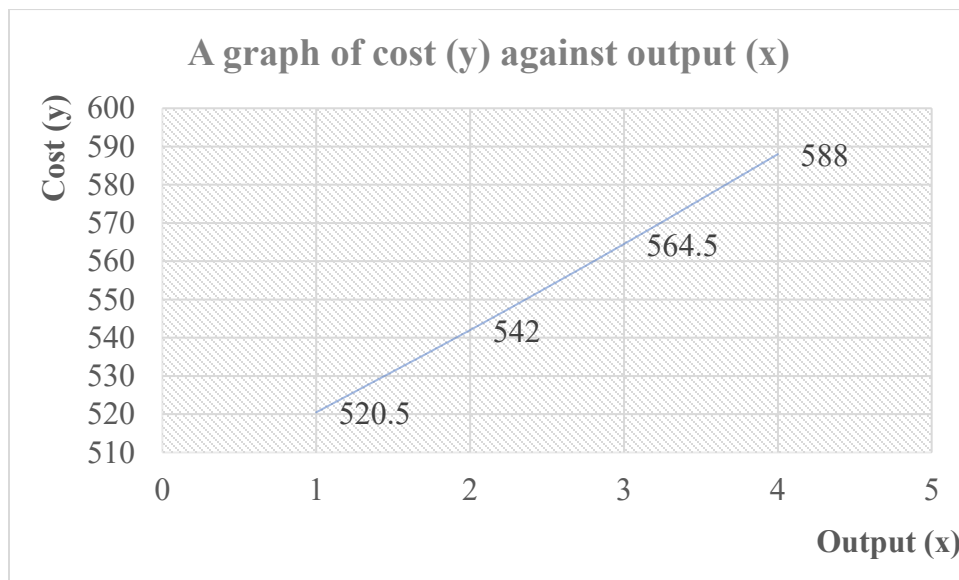
Marginal revenue is derived from total revenue

$$R(x) = 100x - 2x^2$$

$$R'(x) = 100 - 4x$$

iv) **Plotting the graph of the total cost function for all value ranging from 1 to 4 bags.**

$$C(x) = 20x + 0.5x^2 + 500$$



QUESTION THREE

Marking Guide

| Question Three | Description | Marks |
|----------------|---|-----------|
| a) | Index Numbers | |
| | Totals (1 mark each) | 4 |
| | Formula of Laspeyres price index | 1 |
| | Computation of Laspeyres' price index | 1 |
| | Interpretation of Laspeyres price index | 2 |
| | Formula of Paasche's price index | 1 |
| | Computation of Paasche's price index | 1 |
| | Interpretation of Paasche's price index | 2 |
| | Maximum marks | 12 |
| b) | Network Analysis | |
| | Activity (0.5 marks for a well-drawn and labelled activity). Do not award for any activity that does not have an arrow | 7 |
| | Critical path | 0.5 |
| | Project duration | 0.5 |
| | Maximum marks | 8 |
| | Total Marks | 20 |

Model Answer

a) Index numbers

| | 2023 | | 2024 | |
|----------------------|------------------|---------------|------------------|---------------|
| Commodities | Price (FRW) - P0 | Quantity - Q0 | Price (FRW) - P1 | Quantity - Q1 |
| Milk (litres) | 700 | 100 | 900 | 110 |
| Maize Flour (kgs) | 900 | 120 | 950 | 140 |
| Cooking oil (litres) | 1,500 | 220 | 1,550 | 210 |
| Salt (kgs) | 500 | 170 | 600 | 180 |

| Commodities | P0Q0 | P0Q1 | P1Q0 | P1Q1 |
|----------------------|----------------|----------------|----------------|----------------|
| Milk (litres) | 70,000 | 77,000 | 90,000 | 99,000 |
| Maize Flour (kgs) | 108,000 | 126,000 | 114,000 | 133,000 |
| Cooking oil (litres) | 330,000 | 300,000 | 341,000 | 310,000 |
| Salt (kgs) | 85,000 | 90,000 | 102,000 | 108,000 |
| Total | 593,000 | 593,000 | 647,000 | 650,000 |

Laspeyres' Price Index

$$\text{Laspeyres' Price Index} = \frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times 100$$

$$\text{Laspeyres' Price Index} = \frac{647,000}{593,000} \times 100 = 109.11$$

**Interpretation: The prices for the basket of goods increased by 9.11% from 2023 to 2024.
With Laspeyres, prices are under stated**

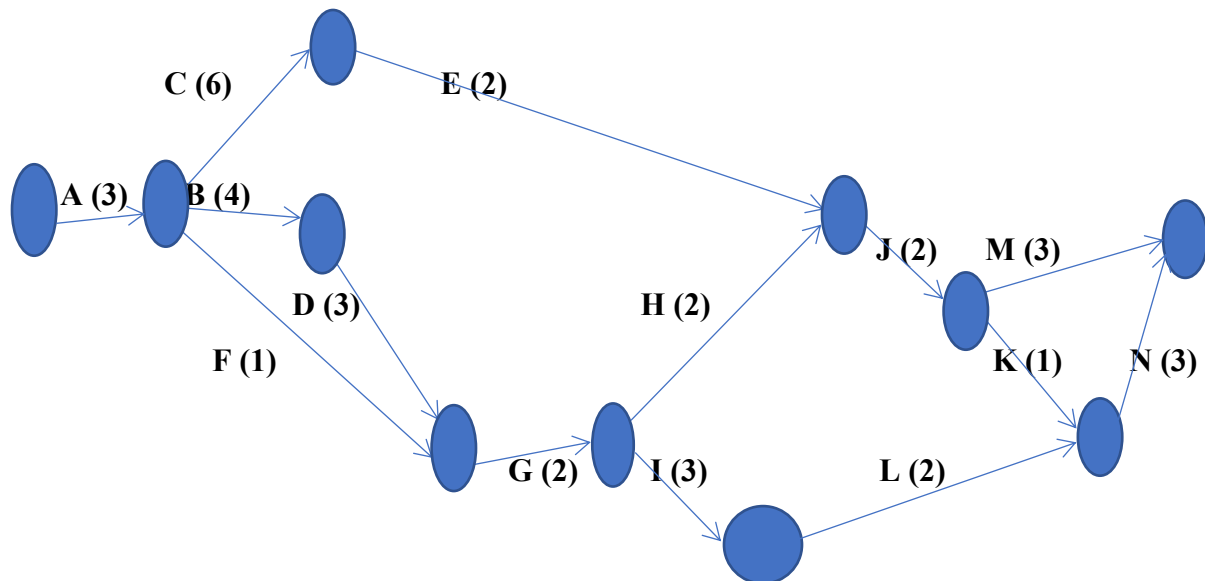
Paasche's Price Index

$$\text{Paasche's Price Index} = \frac{\sum P_1 Q_1}{\sum P_0 Q_1} \times 100$$

$$\text{Paasche's Price Index} = \frac{650,000}{593,000} \times 100 = 109.612$$

**Interpretation: The prices for the basket of goods increased by 9.612% from 2023 to 2024.
With Paasche, prices are over stated**

b) Network diagram for the project



Paths

$$A - C - E - J - M: \quad 3 + 6 + 2 + 2 + 3 = 16 \text{ weeks}$$

$$A - C - E - J - K - N: \quad 3 + 6 + 2 + 2 + 1 + 3 = 17 \text{ weeks}$$

$$A - B - D - G - H - J - M: \quad 3 + 4 + 3 + 2 + 2 + 2 + 3 = 19 \text{ weeks}$$

A – B – D – G – I – L – N: **3 + 4 + 3 + 2 + 3 + 2 + 3 = 20 weeks**

A – F – G – I – L – N: **3 + 1 + 2 + 3 + 2 + 2 = 13 weeks**

The critical path is A – B – D – G – I – L – N

The project duration is 20 weeks

QUESTION FOUR

Marking Guide

| Question Four | Description | Marks |
|---------------|---|-----------|
| a) | Simultaneous Equation | |
| | Formation of equation (1 marks each) | 3 |
| | Rewriting in the matrix form | 1 |
| | Determinant of matrix A | 2 |
| | Determinant of matrix X | 2 |
| | Determinant of matrix Y | 2 |
| | Determinant of matrix Z | 2 |
| | Computation of the values x, y and z (0.5 marks each) | 1.5 |
| | Conclusion for x, y and z (0.5 marks each) | 1.5 |
| | Maximum marks | 15 |
| b) | Net Present Value (NPV) | |
| | Project X (0.5 marks for each of the present values) | 2 |
| | Project Y (0.5 marks for each of the present values) | 2 |
| | Award 1 mark for conclusion | 1 |
| | Maximum marks | 5 |
| | Total Marks | 20 |

Model Answer

a) Solution

Formation of equations

$$x + 2y + 5z = 23$$

$$3x + y + 4z = 26$$

$$6x + y + 7z = 47$$

Re-writing in matrix format

Let this matrix be named A or Matrix A

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 3 & 1 & 4 \\ 6 & 1 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 23 \\ 26 \\ 47 \end{bmatrix}$$

Using determinant method

We find the determinants under each variable and the main determinant of matrix A

Determinant of matrix A

$$\Delta = \begin{vmatrix} 1 & 2 & 5 \\ 3 & 1 & 4 \\ 6 & 1 & 7 \end{vmatrix}$$

$$\Delta = 1(7 - 4) - 2(21 - 24) + 5(3 - 6) = -6$$

Determinant of X

$$23x + 2y + 5z$$

$$26x + y + 4z$$

$$47x + y + 7z$$

$$\Delta_x = \begin{vmatrix} 23 & 2 & 5 \\ 26 & 1 & 4 \\ 47 & 1 & 7 \end{vmatrix}$$

$$\Delta_x = 23(7 - 4) - 2(182 - 188) + 5(26 - 47) = -24$$

Determinant of Y

$$1 + 23y + 5z$$

$$3 + 26y + 4z$$

$$6 + 47y + 7z$$

$$\Delta_y = \begin{vmatrix} 1 & 23 & 5 \\ 3 & 26 & 4 \\ 6 & 47 & 7 \end{vmatrix}$$

$$\Delta_y = 1(182 - 188) - 2(21 - 24) + 5(141 - 156) = -12$$

Determinant of Z

$$x + 2y + 23z$$

$$3x + y + 26z$$

$$6x + y + 47z$$

$$\Delta_z = \begin{vmatrix} 1 & 2 & 23 \\ 3 & 1 & 26 \\ 6 & 1 & 47 \end{vmatrix}$$

$$\Delta_z = 1(47 - 26) - 2(141 - 156) + 23(3 - 6) = -18$$

Computing values of x, y and z

$$x = \frac{\Delta_x}{\Delta} = \frac{-24}{-6} = 4$$

$$y = \frac{\Delta_y}{\Delta} = \frac{-12}{-6} = 2$$

$$z = \frac{\Delta_z}{\Delta} = \frac{-18}{-6} = 3$$

Conclusion: The company is likely to produce 4 chairs, 2 tables and 3 cabinets

b) Solution for NPV

| Year | Project X (FRW) | Project Y (FRW) | PVIF (12%) | Present Values of Project X (FRW) | Present Values of Project Y (FRW) |
|------|-----------------|-----------------|------------|-----------------------------------|-----------------------------------|
| 0 | (5,000,000) | (5,000,000) | 1.000 | (5,000,000) | (5,000,000) |
| 1 | 1,000,000 | 1,500,000 | 0.893 | 893,000 | 1,339,500 |
| 2 | 2,500,000 | 2,000,000 | 0.797 | 1,992,500 | 1,594,000 |
| 3 | 3,000,000 | 2,940,000 | 0.712 | 2,136,000 | 2,093,280 |
| NPV | | | | 21,500 | 26,780 |

Conclusion

Both projects are viable because have positive NPV. However, IMENA Ltd should invest in project Y because has higher NPV than X and IMENA Ltd has only limited initial investment of 5,000,000 FRW.

QUESTION FIVE

Marking Guide

| Question Five | Description | Marks |
|---------------|--|-----------|
| a) | Mugisha Ltd | |
| i) | Venn Diagram | |
| | Definition of sets (0.5 marks each) | 4 |
| | Labelling values in the Venn diagram (0.5 marks each) | 4 |
| | Maximum marks | 8 |
| ii) | Customers who preferred at least 2 products | |
| | Computation of the value of x | 1 |
| | Computation of customers who preferred at least 2 products | 1 |
| | Maximum marks | 2 |
| b) | Forecasting | |
| i) | Linear equation | |
| | Totals (0.5 marks each) | 2.5 |
| | Formula for b | 0.5 |
| | Computation of b | 2 |
| | Formula for a | 0.5 |
| | Computation of a | 2 |
| | Linear equation | 0.5 |
| | Maximum marks | 8 |
| ii) | Sales forecast for 2026 | |
| | Sales forecast equation | 1 |
| | Sales forecast for 2026 | 1 |
| | Maximum marks | 2 |
| | Total Marks | 20 |

Model Answer

a) Solution

i) Representation of the following information on a Venn Diagram

Let B be the customers who prefer Boneza product, K the customers who prefer Kinigi product and N be the customers who prefer Ndama product.

Definition of sets

$$n(B \cup K \cup N) = 950$$

$$n(B' \cap K \cap N) = 30$$

$$n(B \cap K \cap N') = 70$$

$$n(B \cap K \cap N) = 80$$

$$n(B \cap K' \cap N) = x$$

$$n(B \cap K' \cap N') = x$$

$$n(B) = 470$$

$$n(K) = 300$$

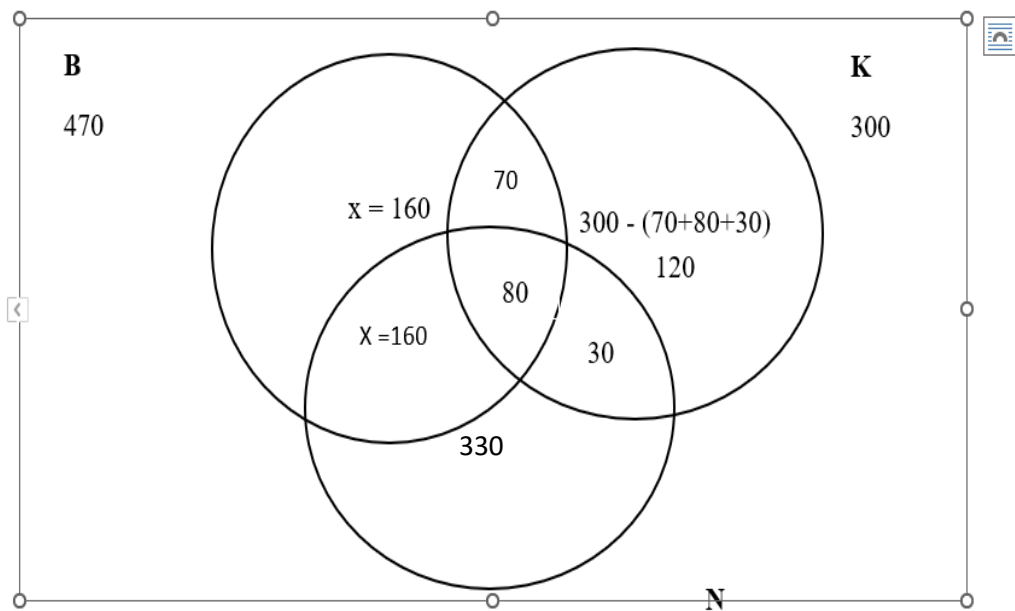
Ndama

$$160 + n + 120 + 70 + 30 + 160 + 80 = 950$$

$$\text{Only Ndama}(n) = 330$$

A Venn Diagram representing the data provided in the question

A =



ii) Calculate the number of customers who preferred at least two products. (2 Marks)

The number of customers that prefer at least two products

First find the value of x

$$n(B) = 470$$

$$n(B) = x + x + 80 + 70$$

$$470 = x + x + 80 + 70$$

$$2x = 320$$

$$x = 160$$

The number of customers that prefer at least two products are $n(B \cap K \cap N') + n(B' \cap K \cap N) + n(B \cap K' \cap N) + n(B \cap K \cap N)$

The number of customers that prefer at least two products = $70 + 30 + 160 + 80 = 340$

b) Forecasting

i) Finding the linear equation

Solution

| Year | Sales (“Millions”) FRW, y | x | x ² | xy |
|----------|---------------------------|-----------|----------------|-------------|
| 2018 | 65 | 1 | 1 | 65 |
| 2019 | 88 | 2 | 4 | 176 |
| 2020 | 132 | 3 | 9 | 396 |
| 2021 | 190 | 4 | 16 | 760 |
| 2022 | 275 | 5 | 25 | 1375 |
| 2023 | 300 | 6 | 36 | 1800 |
| 2024 | 320 | 7 | 49 | 2240 |
| 7 | 1370 | 28 | 140 | 6812 |

The linear regression equation is given by $y = a + bx$

Therefore, we have to first find constant values a and b

$n=7$, $\sum y = 1,370$, $\sum x = 28$, $\sum x^2 = 140$, $\sum xy = 6,812$

$$b = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2} = \frac{7 \cdot 6,812 - 28 \cdot 1,370}{(7 \cdot 140) - (28)^2} = \frac{47,684 - 38,360}{980 - 784} = \frac{9,324}{196} = 47.57$$

$$a = \frac{\sum y - b \sum x}{n} = \frac{1,370 - 47.57 \cdot 28}{7} = \frac{38.04}{7} = 5.43$$

$$y = 5.43 + 47.57x$$

ii) Forecast the sales of the company for the year 2026.

Sales, $y = 5.43 + 47.57x$

$x = 9$ from 2018 to 2026 (9 years)

Sales, $y = 5.43 + 47.57 \cdot 9 = 433.56$

The value forecasted sales for 2026 is FRW 433,560,000

QUESTION SIX

Marking Guide

| Question Six | Description | Marks |
|--------------|---------------------------|-------|
| | Karungi Ltd | |
| a) | Hypothesis Testing | |

| | | |
|-----------|--|-----------|
| | Computation of mean | 1 |
| | Computation of standard deviation | 1 |
| | Stating a null hypothesis | 1 |
| | Stating an alternative hypothesis | 1 |
| | Computation of standardized value | 1 |
| | Computation of critical value | 1 |
| | Decision | 2 |
| | Maximum marks | 8 |
| b) | Difference between simple random sampling and stratified sampling | |
| | Definition of simple random sampling | 1 |
| | Definition of stratified sampling | 1 |
| | Advantages of simple random sampling (1 mark each) | 2 |
| | Maximum marks | 4 |
| c) | Explanation of methods of data presentation | |
| | Methods (1 mark each) | 4 |
| d) | Characteristics of measures of dispersion | |
| | Characteristics (1 mark each) | 4 |
| | Total Marks | 20 |

Model Answer

a) Conducting a hypothesis test

Solution

Computation sample mean

$$\text{Mean, } \bar{x} = \frac{\sum x}{n} = \frac{310+298+295+303+292+288+300}{7} = \frac{2,086}{7} = 298$$

Computation of sample standard deviation

$$\text{Standard deviation, } s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{(310-298)^2 + (298-298)^2 + (303-298)^2 + (292-298)^2 + (288-298)^2 + (300-298)^2}{7-1}} = \sqrt{\frac{318}{6}} = \sqrt{53} = 7.28$$

Statement of hypotheses

Null hypothesis $H_0: \mu = 300$ (New machine produces 300 bottles per day)

Alternative hypothesis $H_A: \mu \neq 300$ (New machine does not produce exactly 300 bottles per day)

Standardized value

$$t = \frac{\frac{x_i - \mu}{s}}{\frac{s}{\sqrt{n}}} = \frac{\frac{298 - 300}{7.28}}{\frac{7.28}{\sqrt{7}}} = \frac{-2}{2.65} = \frac{-2}{2.75} = -0.73$$

Critical value

Critical t – value (two – tailed, $\alpha = 0.05$) degree of freedom $df = n - 1 = 7 - 1 = 6$

From the t – distribution table $t_{\text{critical}} = \pm 2.447 \approx \pm 2.45$

Decision

Since -2.45 is less than -0.73 , we fail to reject the null hypothesis. Therefore, we support the claim that the new machine produces 300 bottles of juice per day at 5% level of significance.

b) Difference between simple random sampling and stratified sampling techniques

Simple random sampling technique is the most basic method among the probability sampling techniques. This method uses a sampling frame that lists all members of the target population, giving each member an equal chance of being selected in the sample.

While

Stratified sampling is a sampling technique applied to a heterogeneous (having groups with varied attributes) population that can be subdivided into homogeneous (with manufactures television sets, radios and cameras). These are homogeneous groups referred to as strata; hence, the name stratified sampling.

Stratified sampling, therefore, involves obtaining simple samples from each of the strata of the population and the simple samples combined to give the stratified sample of the population. This ensures that each homogeneous segment of the population is proportionally represented in the sample.

Advantages of simple random sampling

- It eliminates bias because every member of the population has equal chance of being considered.
- The sample is always representative of the population.
- The sample is generated with ease.

c) Explanation of methods of data presentation

- **Tables:** A table is a layout of information or data in form of columns and rows. They are the simplest form of data display. One of the forms of data presentation is using tables just as the table provided in the question having a column for days and another column for the bottles produced per day
- **Pie Charts:** Pie charts are a way of representing data so that it is easier to understand and interpret. A pie chart shows the totality of the data being represented using sectors of circle. The circle is split into sectors; each one being drawn in proportion to

the data it represents. A whole circle is 360 degrees (360°) so the sectors of a pie chart will be fractions of the 360° . The same data can be presented on a pie chart.

- **Simple Bar Charts:** In a simple bar chart, a separate bar for each class is drawn to a height proportional to the class frequency. You can identify the figure that each bar represents at the base of the bar or you can use a key to show that a color or shade indicates a particular item. The data provided in the question can be displayed on a bar chart.
- **Line Graphs:** A line graph represents the data concerning one variable on the horizontal and other variable on the vertical axis. It uses points and lines to show change over time. It is plotted from a set of points and then joined by a line. Different data sets can be plotted on the same graph but a key must be used to identify each data set. A line graph could be used to present Karungi Ltd.'s data.

d) Characteristics of measures of dispersion

- Simple to comprehend
- Easy to compute
- Specifically defined, using all the items in the distribution for its computation
- Easily used in further algebraic calculations
- Unaffected drastically by the extreme items in the distribution
- Affected as little as possible by fluctuations in sampling.

QUESTION SEVEN

Marking Guide

| Question Seven | Description | Marks |
|-----------------------|---|--------------|
| | Neema Ltd | |
| a) | Decision Theory | |
| i) | Computation of expected values under certainty | 1 |
| | Computation of EMV for each product (1 mark each) | 3 |
| | Maximum marks | 4 |
| ii) | Maximax, Maximin and Hurwitz | |
| | Maximax (0.5 marks for decision and 0.5 marks for the reason - value) | 1 |
| | Maximin (0.5 marks for decision and 0.5 marks for the reason - value) | 1 |
| | Hurwitz (1 mark for each payoff computed) | 3 |
| | Decision for Hurwitz | 1 |
| | Maximum marks | 6 |
| b) | IGIRE Ltd | |
| i) | Probability of exactly 4 batteries | 2 |
| ii) | Probability of 0 | 1 |
| | Probability of 1 | 1 |
| | Probability of 2 | 1 |
| | The total of the P (0), P (1) and P (2) | 1 |
| | Maximum marks | 4 |
| c) | Properties of probability | |
| | Properties of probability (1 mark each) | 4 |
| | Total Marks | 20 |

Model Answer

- a) Neema Limited
- i) **Determination of the expected value of perfect information using expected monetary values and interpretation of**

| | Maximax | Maximin | Hurwitz | EMV | EV, Certainty | EVPI |
|------------------|------------------------|------------------------|----------------|----------------|----------------------|---------------|
| | Maximum payoffs | Minimum payoffs | Payoffs | Payoffs | | |
| Product A | 500,000 | 150,000 | 430,000 | 352,500 | 404,500 | 42,500 |
| Product B | 420,000 | 200,000 | 376,000 | 362,000 | | |
| Product C | 400,000 | 250,000 | 370,000 | 340,000 | | |

Solution

Determination of expected value of perfect information using expected monetary value method

Expected values under certainty

Expected values under certainty = $(500,000 \times 0.20) + (420,000 \times 0.35) + (400,000 \times 0.30) + (250,000 \times 0.15) = \text{FRW } 404,500$

Expected Monetary Values (EMV)

Product A; $EMV = (500,000 \times 0.20) + (400,000 \times 0.35) + (300,000 \times 0.30) + (150,000 \times 0.15) = \text{FRW } 352,500$

Product B; $EMV = (400,000 \times 0.20) + (420,000 \times 0.35) + (350,000 \times 0.30) + (200,000 \times 0.15) = \text{FRW } 362,000$

Product C; $EMV = (300,000 \times 0.20) + (350,000 \times 0.35) + (400,000 \times 0.30) + (400,000 \times 0.15) = \text{FRW } 340,000$

The maximum EMV is **FRW 362,000**

Expected Value of Perfect Information (EVPI)

$EVPI = \text{expected value under certainty} - \text{expected monetary value (maximum)}$

$EVPI = \text{FRW } 404,500 - \text{FRW } 362,000 = \text{FRW } 42,500$

This is the amount of money the company is expected to pay so as to acquire perfect information for accurate decision making.

- i) **Determination of the appropriate decision for Neema Limited under Maximax, Maximin and Hurwitz criteria.**

Maximax Criterion

Decision: The decision under Maximax Criterion is to launch Product A first because it has the highest maximum payoff of FRW 500,000

Maximin Criterion

Decision: The decision under Maximin Criterion is to launch Product C first because it has the highest minimum payoff of FRW 250,000

Hurwitz Criterion

$$\text{Payoff} = \alpha * \text{Maximum Value} + (1 - \alpha) * \text{Minimum Value} \quad \alpha = 0.8$$

$$\text{Product A; Payoff} = (0.8 * 500,000) + (0.2 * 150,000) = \text{FRW } 430,000$$

$$\text{Product B; Payoff} = (0.8 * 420,000) + (0.2 * 200,000) = \text{FRW } 376,000$$

$$\text{Product C; Payoff} = (0.8 * 400,000) + (0.2 * 250,000) = \text{FRW } 370,000$$

Decision: The decision under Hurwitz Criterion is to launch Product A first since it has the highest maximum payoff of FRW 430,000

b) IGIRE Ltd

i) **Probability of exactly 4 batteries**

Solution

$$P(X=x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$p = 5\% = 0.05, q = 1 - 0.05 = 0.95, n = 25, x = 4$$

$$P(x=4) = \frac{25!}{4!(25-4)!} 0.05^4 (1 - 0.05)^{25-4}$$

$$P(x=4) = \frac{25!}{4!21!} 0.05^4 0.95^{21} = 0.027 = 2.7\%$$

ii) **Probability of at most 2 batteries**

Solution

$$P(x \leq 2) = P(x=0) + P(x=1) + P(x=2)$$

$$P(x=0) = \frac{25!}{0!25!} 0.05^0 0.95^{25} = 0.277$$

$$P(x=1) = \frac{25!}{1!24!} 0.05^1 0.95^{24} = 0.365$$

$$P(x=2) = \frac{25!}{2!23!} 0.05^2 0.95^{23} = 0.231$$

$$P(x \leq 2) = 0.277 + 0.365 + 0.231 = 0.873$$

Required:

c) Properties of probability

- **Probability limits**

The probability of an event must lie between 0 to 1. A probability close to 0 indicates that the event is not likely to occur. A probability close to one indicates that the event is most likely to happen.

- **Complementary events**

The complement of an event A is the event that A does not occur. This is denoted as A^1 or A^c . If the probability of A is $P(A)$ then the $P(A^1) = 1 - P(A)$.

Example: The probability of a product being of acceptable quality is $2/3$, therefore, the probability of a product not being of acceptable quality = $1 - 2/3 = 1/3$.

- **Exhaustive events**

These are events whose sum of the probabilities of all possible outcomes in a sample space is one.

Example: When a die is thrown once, the possible outcomes are 1, 2, 3, 4, 5, 6.

$$P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1/6 + 1/6 + 1/6 + 1/6 + 1/6 + 1/6 = 6/6 = 1$$

- **Independent events** These are events where the occurrence of one does not affect or influence the occurrence of the others. For example, the performance of female students and the male students in an examination.

For two independent events A and B the probability that both events occur is written as $P(A \text{ and } B)$ or $P(A \cap B)$.

By the multiplication rule $P(A \cap B) = P(A) \times P(B)$

Example: If two events A and B are independent such that $P(A) = 0.35$ and $P(B) = 0.6$, find the probability that both events occur. $P(A \cap B) = P(A) \times P(B) = 0.35 \times 0.6 = 0.21$

End of Marking Guide and Model Answers